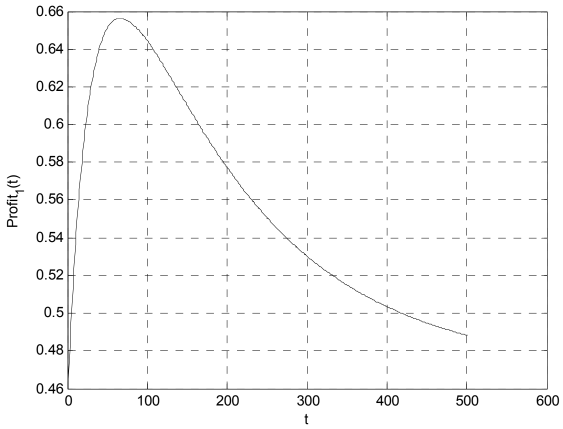


**Figure 2.** The profit of the first firm (t, π1(t))

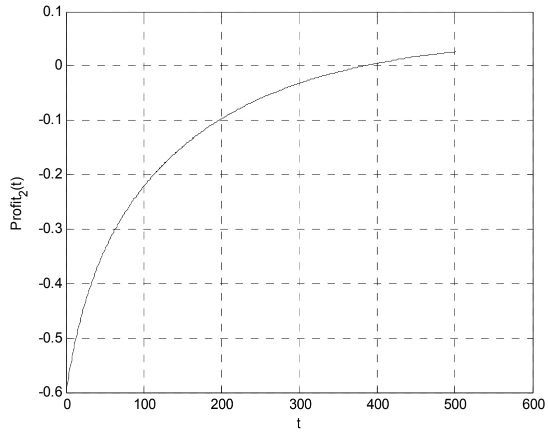


**Figure 3.** The profit of the second firm (t, π2(t))

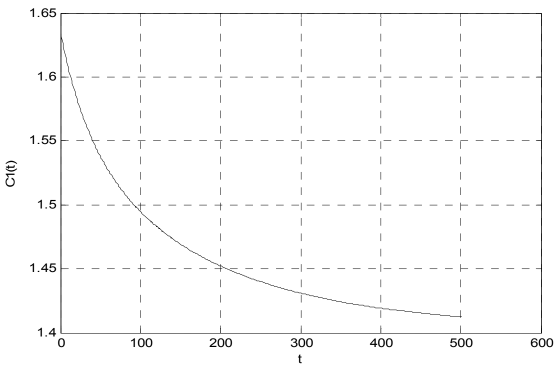


**Figure 4.** The cost of the first firm (t, C1(t))

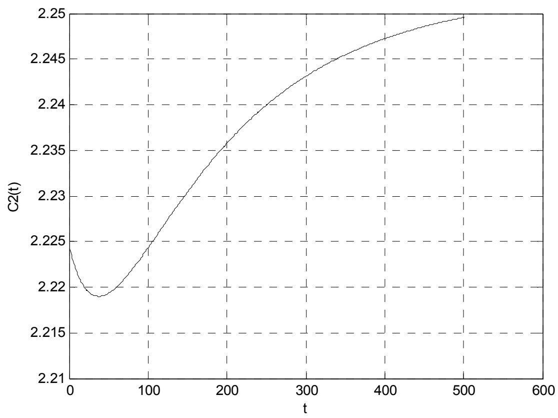


**Figure 5.** The cost of the second firm (t, C2(t))

## 5. Conclusions

In this paper, the dynamics of a nonlinear duopoly game with bounded rationality and distributed time delay is analyzed. The game is described by a nonlinear differential system with two equations which contain distributed time delays, because the decisions made by the economic agents at time t depend on the past decisions.

For two types of kernels, weak and Dirac, the stability of equilibria and bifurcation are investigated. It is found that if there is no delay the equilibrium point is locally asymptotically stable. If $k_1(s) = d_1 e^{-d_1 s}$, $k_2(s) = \delta(s - \tau_2)$, with $d_1 = d_{10} > 0$, $\tau_2 > 0$, there exists a value $\tau_{20}$

for which a Hopf bifurcation occurs and the orbits of system (8) has a limit cycle. If $k_1(s)=\delta(s-\tau_1), k_2(s)=d_2e^{-d_2s},$ with $\tau_1>0, d_2=d_{20}>0,$ there exists a value $\tau_{10}$ for which a Hopf bifurcation occurs and the orbits of system (8) has a limit cycle.

A similar analysis will be carried out for $k_1(s)=d_1e^{-d_1s}$, $k_2(s)=d_2e^{-d_2s}$ and other types of kernels: strong, uniform, or Gauss. Because the parameters of the real models are subject to perturbations, then in our next paper we will consider the stochastic model with distributed time delay.

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